Thomas Harriot and the Mercator map

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Mostly based on Jon V. Pepper, ‘Harriot’s Calculation of Meridional Parts,’ *Archive for History of Exact Sciences*, 1968.

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Flying from New York to London

The shortest route follows a great circle

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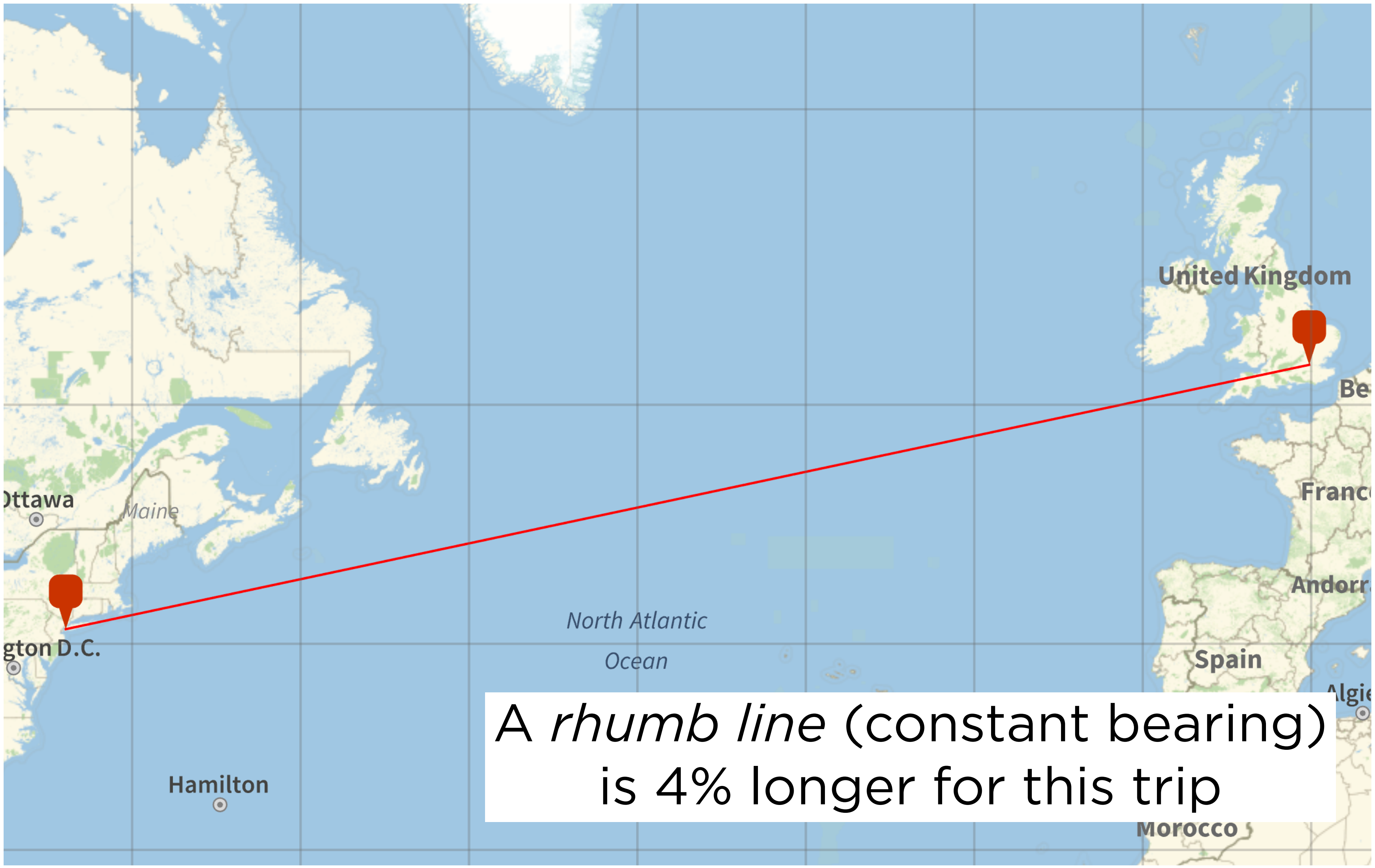
Plotting it on a map

On a map, the great circle route looks wrong

Can’t we just go straight there?



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Continuing the rhumb line

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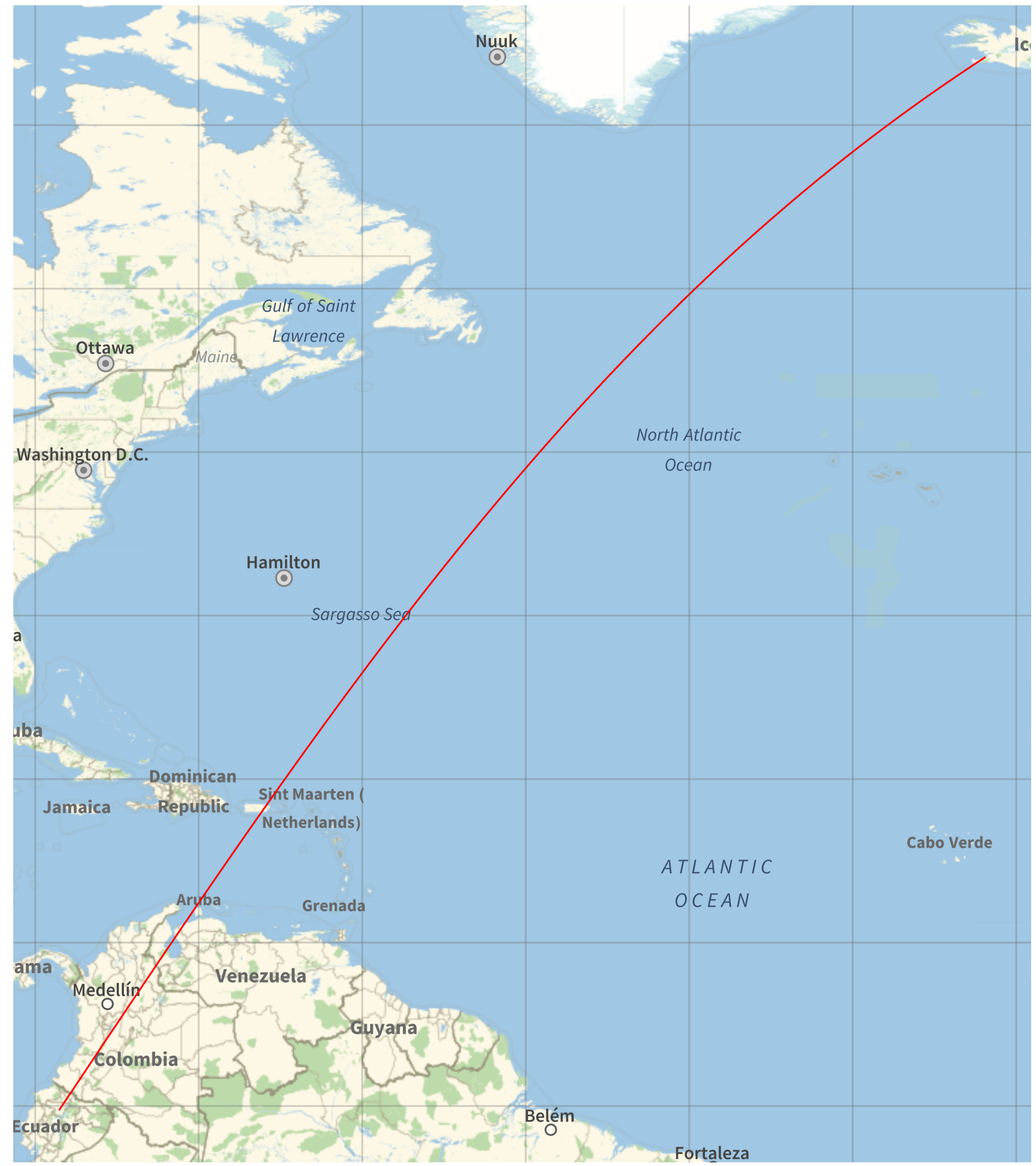
Making a map

On a *plane chart*, rhumb lines appear curved.

Worse still, measured bearings are wrong.

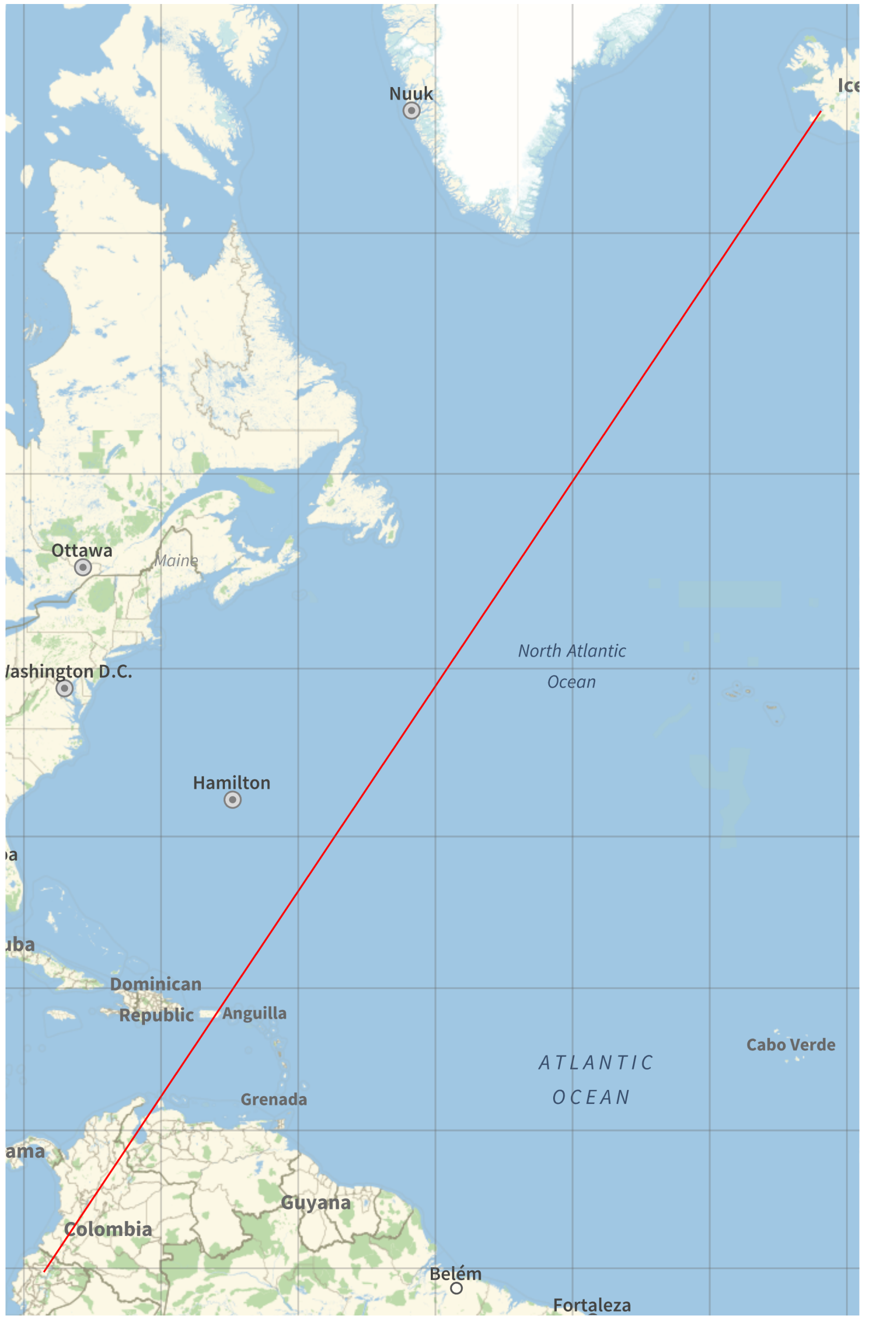


Gerardus Mercator (1512-94)

Huge world map of 1569, made from 18 sheets pasted together.

Stretched vertically at higher latitudes so as to make rhumb lines straight.

But how much? Mercator seems to have used empirical methods.



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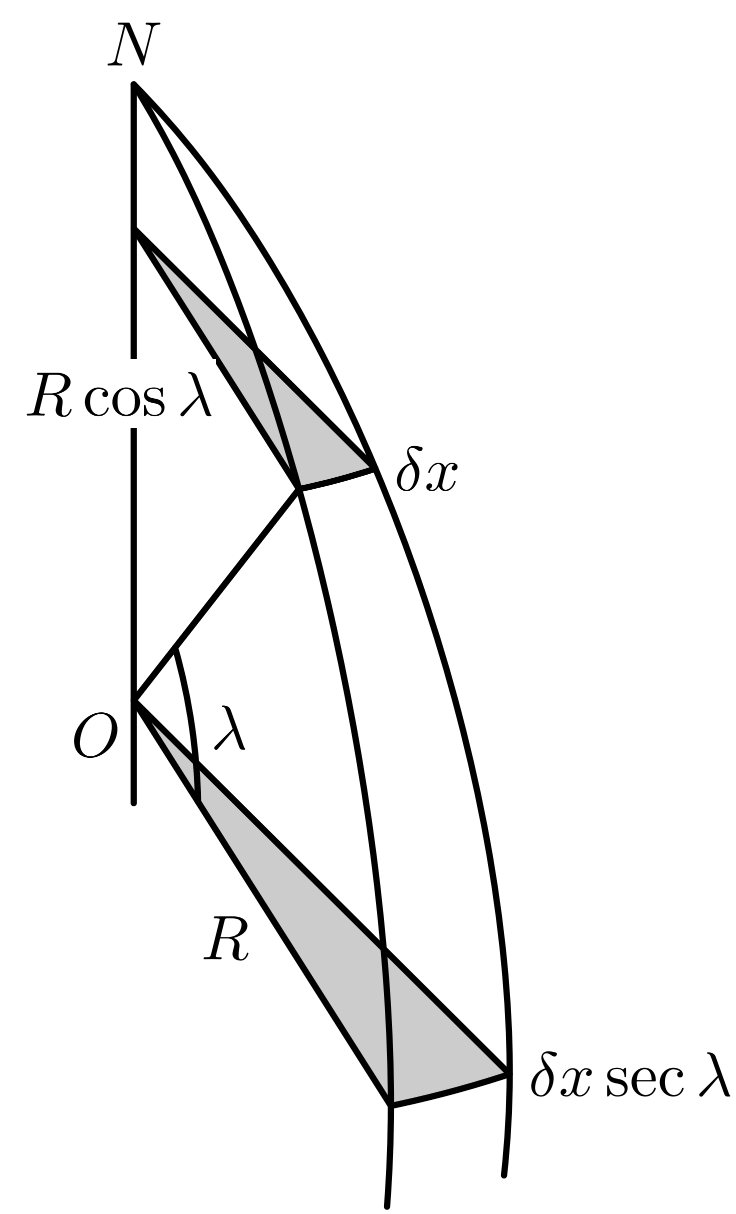
Thomas Harriot (c.1560-1621)

- Undergraduate at St Mary's Hall 1577–80.
- Protégé of Walter Raleigh, and later Henry Percy, Earl of Northumberland.
- Mostly known for his work in algebra.
- Participant in early colony in North America.
- Left behind 1000's of pages of *largely unexplained* calculations.

It's all about the secants

At latitude λ, horizontal distances on the chart are magnified by $\sec\lambda$ to make the meridians parallel.

So locally the vertical distances should be magnified by $\sec\lambda$ also to make the projection angle-preserving.



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A bit of calculus?

$$M(\lambda) = \int_0^\lambda \sec\lambda\, d\lambda = \log\frac{1+t}{1-t} = \log\tan(\frac{\lambda}{2} + \frac{\pi}{4})$$

where $t = \tan\frac{\lambda}{2}$

Sadly, no: neither $\int$ nor $\log$ were in Harriot's vocabulary in 1614.

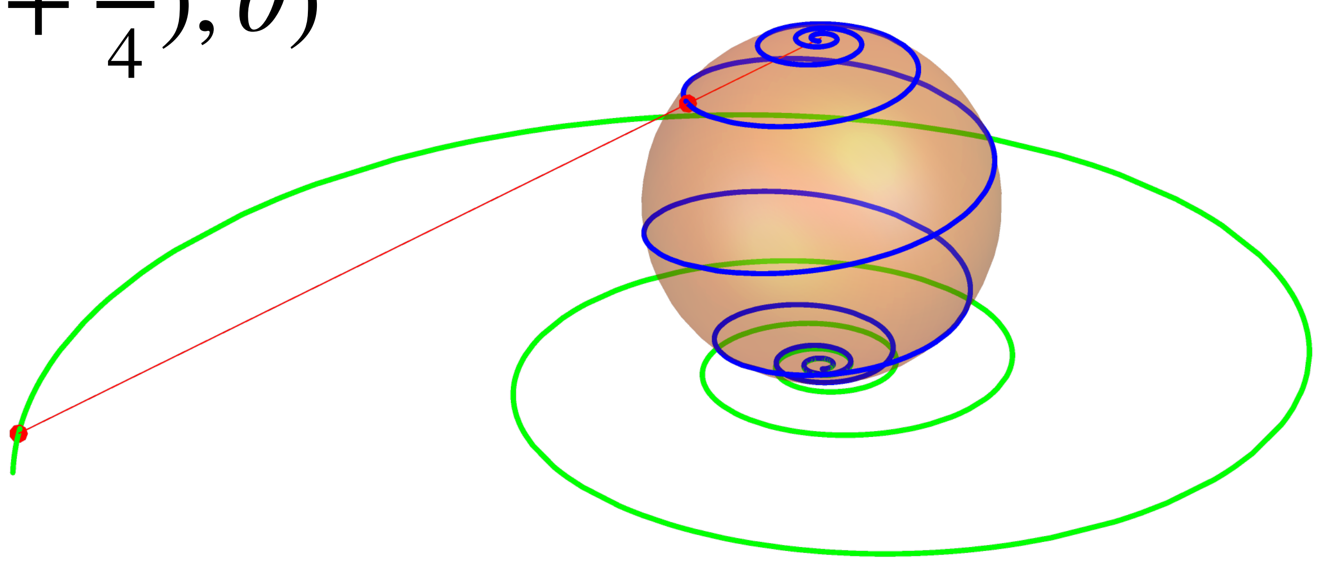
$M(\lambda)$ is the longitude reached at latitude λ on a 45° rhumb.

Stereographic projection

Project from one pole to a plane tangent at the other pole.

Harriot produced a proof this is angle-preserving (a fact already known).

$$(r, \theta, \lambda) \mapsto (r \tan(\frac{\lambda}{2} + \frac{\pi}{4}), \theta)$$



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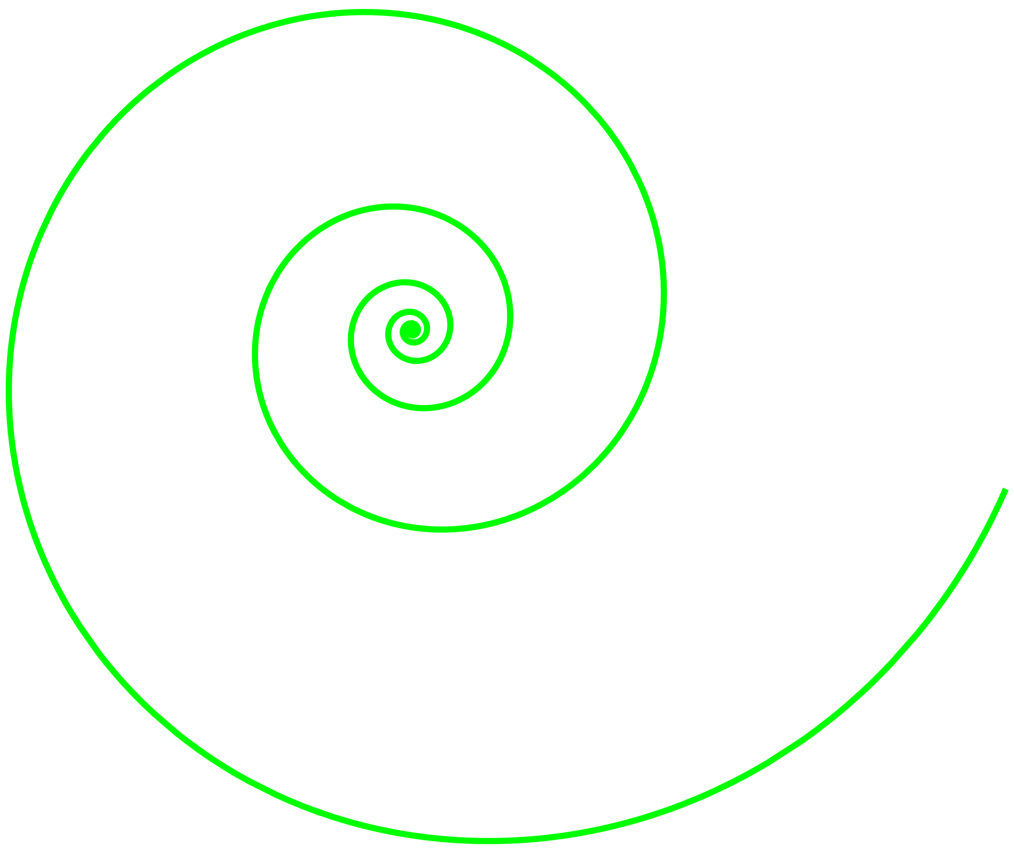
Equiangular spiral

The image of a rhumb line is an equiangular (or "logarithmic" spiral).

The radius increases by a constant ratio as it sweeps out equal angles.

$$r_n/r_o = (r_1/r_0)^n$$



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The fundamental equation

Let $\beta < 1$ be the ratio r_1/r_0 for a $45°$ spiral and an angle of (say) $\theta = 1'$. If

$$\tan(\frac{\pi}{4} - \frac{\lambda}{2}) = \beta^n$$

then $M(\lambda) = n\theta$. [Actually $\beta = e^{-\theta}$.]

To find $M(\lambda)$ for any latitude λ, determine n such that

$$\beta^{n+1} < \tan(\frac{\pi}{4} - \frac{\lambda}{2}) \leq \beta^n$$

then $n\theta \leq M(\lambda) < (n+1)\theta$; use linear interpolation.

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A refinement

Find β as above, then $\alpha = \beta^{60}$ and $\gamma = \beta^{1/100}$.

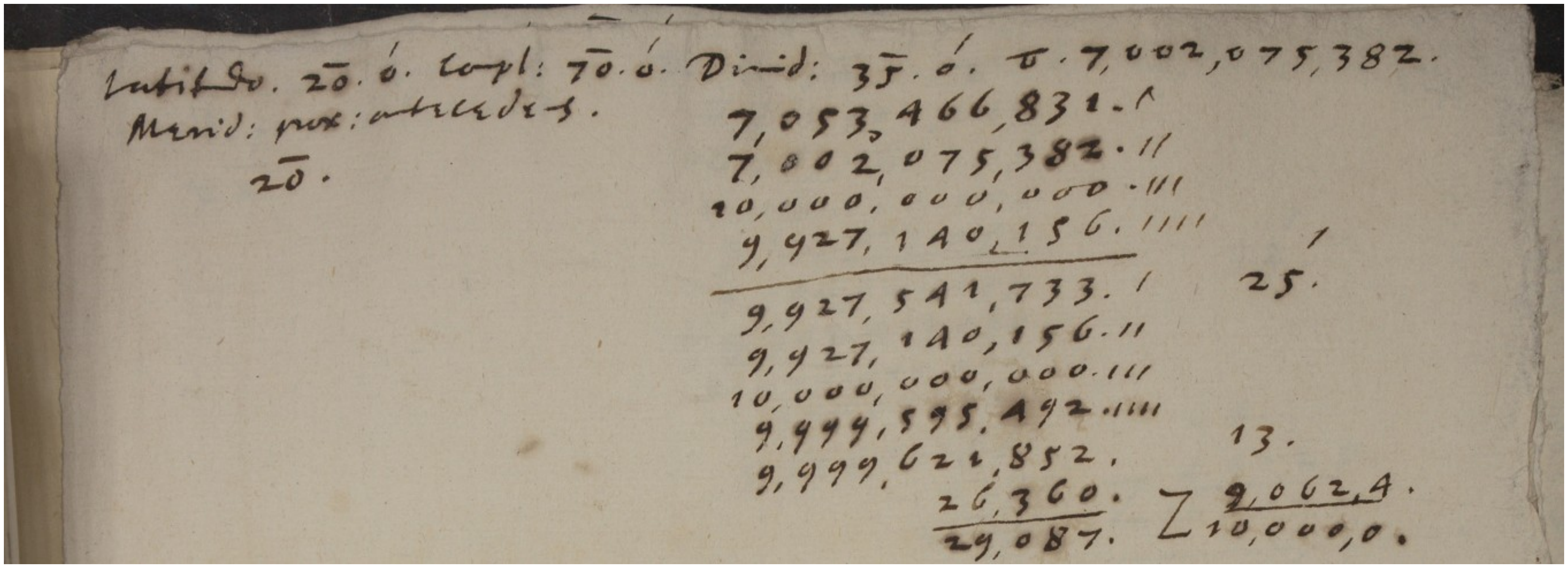
Make tables of powers up to α^{90} and beyond, β^{60} and γ^{100}.

Now if

$$\tan(\frac{\pi}{4} - \frac{\lambda}{2}) \approx \alpha^a \beta^b \gamma^c$$

then $M(\lambda) \approx a^\circ\,(b + c/100)'$ gives degrees and minutes to two decimal places.

Calculating $M(20°)$



$$M(20°) = 20° \, 25'.13\,90624$$

Dozens of pages with such calculations exist!

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In modern notation

Latitude 20°; complement 70°; halved 35°

tan 35°	= 0.7002075382
α^{20}	= <u>0.7053446831</u>
quotient	= 0.9927140156
β^{25}	= <u>0.9927541733</u>
quotient	= 0.9999595492
γ^{13}	= <u>0.9999621852</u>
difference	= 26360
$\gamma^{13}-\gamma^{14}$	= <u>29087</u>
quotient	= 0.90624

$M(20°)$ = 20° 25’.13 90624

Let's check that!

$M(20°)$ calculated as $20° \, 25'.13\,90624$

$$
\begin{aligned}
-\log\tan 35° &= 0.3563785047 \\
&= 20.418984230° \\
&= 20°\,25'.13\,90538
\end{aligned}
$$

The small error is largely explained by the approximate value Harriot used for β.

Harriot's table of meridional parts remained unsurpassed for 350 years.

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Traditional sailing vessels

- Steer on points of compass: N, N by E, NNE, NE by N, … (11¼° apart), and not very accurately.
- Couldn't steer closer than six or seven points to the wind.
- Tacking was a highly skilled manoeuvre for both officers and crew, so usual to gybe.

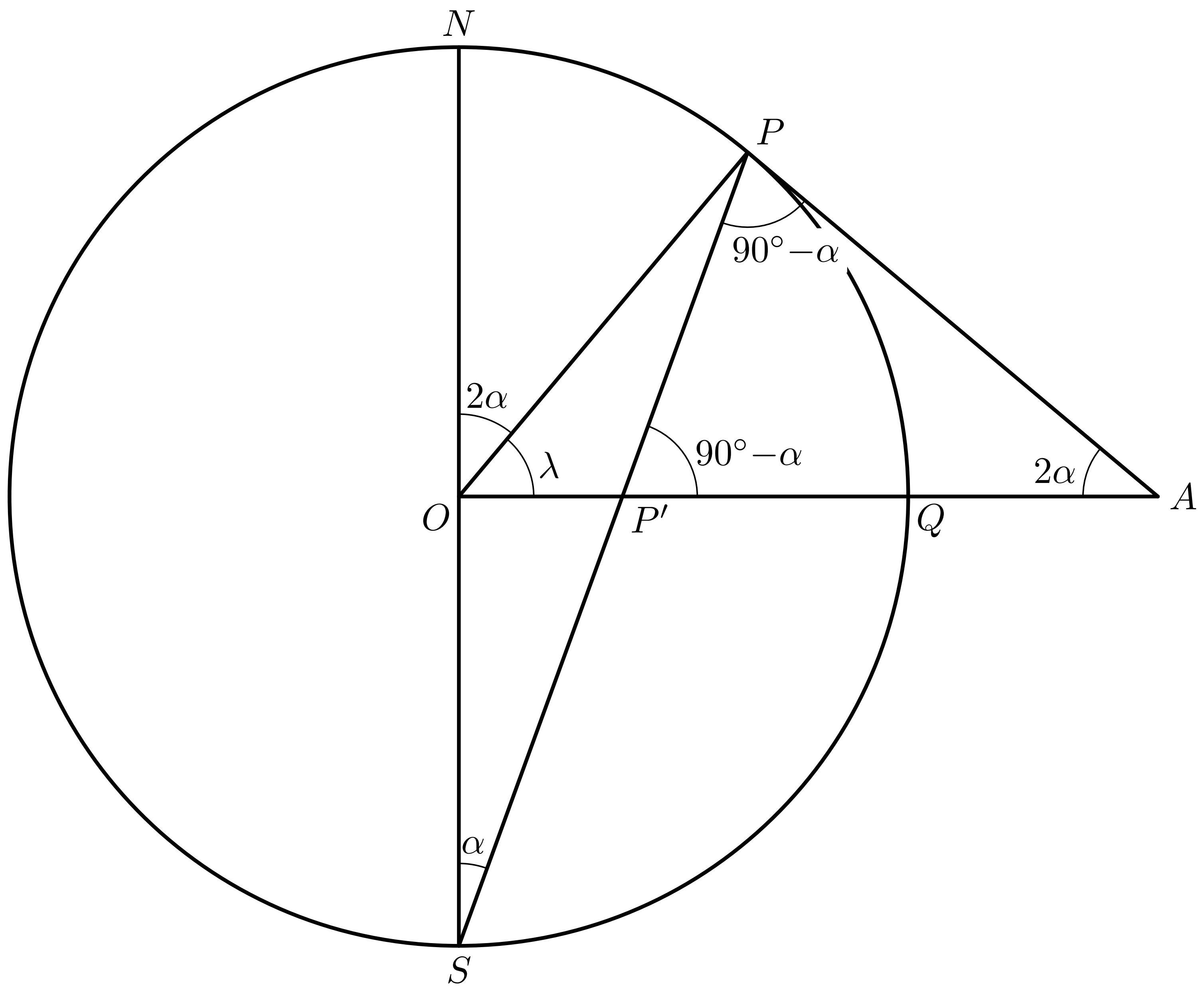
… so it's a puzzle why such precise navigation tables were wanted.

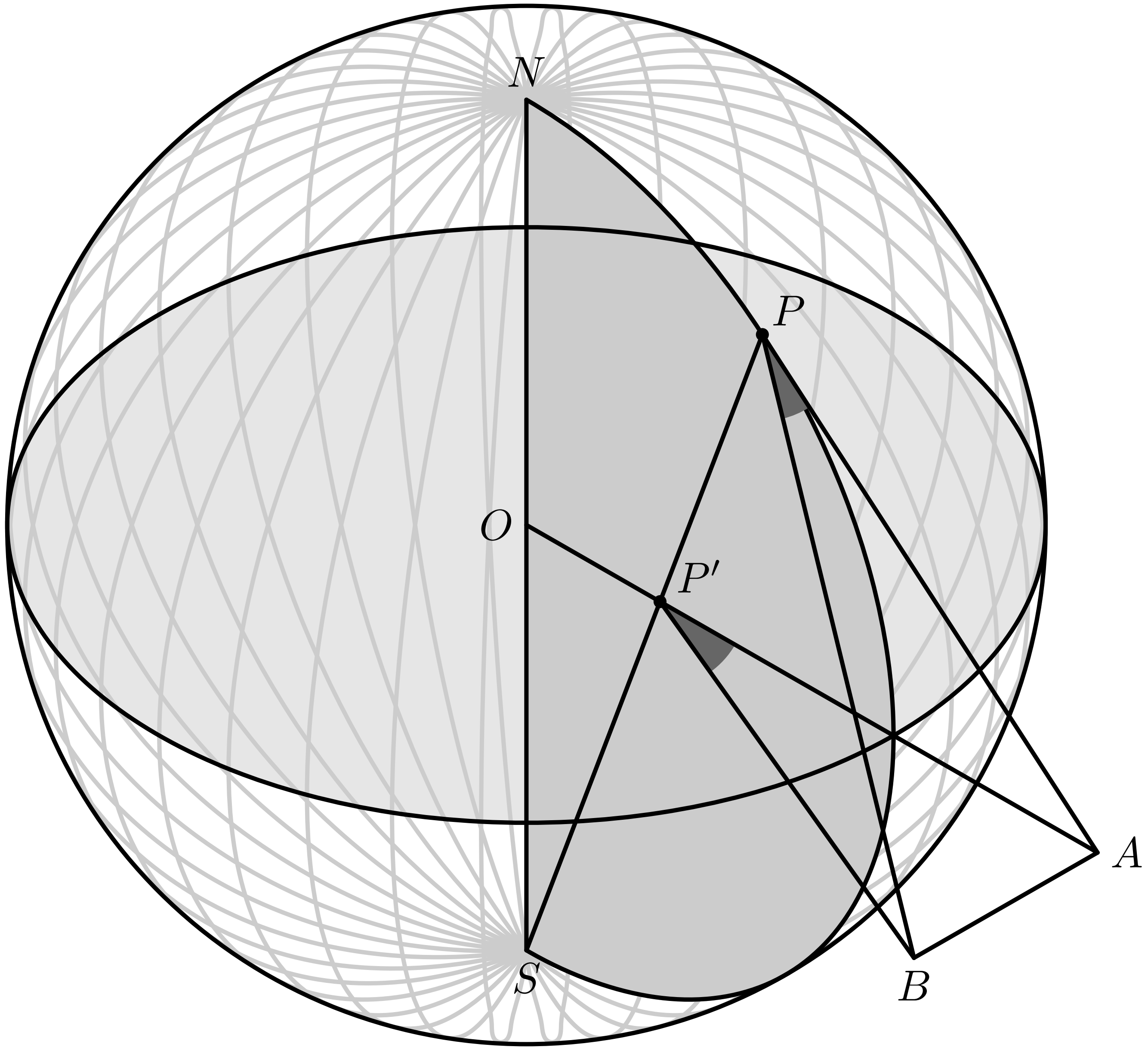
No time to mention

- Additional corrections used at high latitudes (where the distortion becomes extreme).
- Use of second and higher order finite differences for interpolation.
- Algorithms for exponentiation based on binary representation.
- Interval arithmetic to monitor accuracy of extended calculations.

If you want to know more

- Annual Thomas Harriot lecture at Oriel, with past lectures in three volumes (ed. Robert Fox).
- Jon V. Pepper's 1968 paper repays close study.
- Harriot's manuscripts now all collated and online at [Google Harriot manuscripts online].


N
P
90°−α
2α
λ
90°−α
2α
A
O
P′
Q
α
S


N
P
O
P′
A
B
S

The School of Night

… a group of savants known as the School of Night, or by some the School of Atheism, who were very interested in mathematics, the new astronomy, and various kinds of occultism. The School is said to have included Raleigh and 'the wizard earl' of Northumberland, …. But the chief brain of the group is said to have been Thomas Hariot, a dependant of Raleigh's and a man of versatile genius – navigator, astronomer, maker of horoscopes, and an early smoker. The existence of a School of Night, so called, [is debatable]. Nevertheless such groups of students did exist. They had a reputation for far-out learning but also for pedantry and dullness, and they were therefore disliked by more sprightly men who were more interested in wit and poetry.

Frank Kermode, *The Age of Shakespeare*, London, Weidenfeld and Nicolson, 2004, pp. 68–69.

Colophon

- Maps and plots made with Mathematica.
- Geometric diagrams drawn with Metapost.
- PDF fragments combined with Keynote.